TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1,2
Test 2 2021

Section 1 Calculator Free
Vectors

STUDENT'S NAME SOLUTIONS

DATE: Wednesday 31st March **TIME:** 22 minutes **MARKS:** 22

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

Given the following vectors $\underset{\sim}{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\underset{\sim}{c} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, determine the value of each of the following if possible. If not possible give a valid reason.

(a) $\underset{\sim}{a} \bullet (\underset{\sim}{b} + \underset{\sim}{c})$ [2]

$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \left(\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix} \right)$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$= 1 - 6$

$= -5$

(b) $\underset{\sim}{a} + (\underset{\sim}{b} \bullet \underset{\sim}{c})$ [2]

NOT POSSIBLE TO ADD A VECTOR AND A SCALAR

(c) $|\underset{\sim}{a} + \underset{\sim}{b}| + (\underset{\sim}{a} \bullet \underset{\sim}{b})$ [3]

$\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} \right| + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$= \left| \begin{matrix} -2 \\ 4 \end{matrix} \right| + (-3) + 4$

$= \sqrt{20} + 1$

2. (4 marks)

The points A, B and C have position vectors $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ respectively. Determine whether the three points are collinear.

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

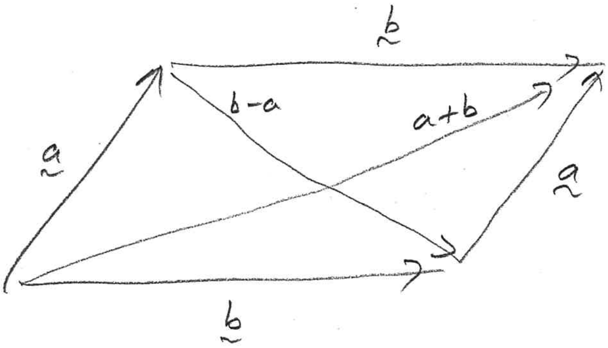
$$\overrightarrow{BC} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$
$$= \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \neq \lambda \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$\therefore$ NOT COLLINEAR

3. (5 marks)

Use vectors to prove the sum of the squares of the lengths of the sides of a parallelogram is equal to the sum of the squares of the lengths of the diagonals.



SIDES $|a|^2 + |b|^2 + |a|^2 + |b|^2$

$= 2|a|^2 + 2|b|^2$

DIAGONALS $|a+b|^2 + |b-a|^2$

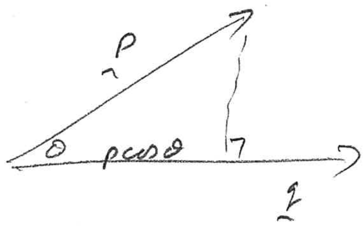
$= |a|^2 + 2|a||b| + |b|^2 + |b|^2 - 2|a||b| + |b|^2$

$= 2|a|^2 + 2|b|^2$

$\therefore$ SUM OF SQUARES SIDES = SUM OF SQUARES OF DIAGONALS

4. (6 marks)

Given $\underset{\sim}{p} = 3\underset{\sim}{i} + 7\underset{\sim}{j}$ and $\underset{\sim}{q} = -2\underset{\sim}{i} + 6\underset{\sim}{j}$, determine



(a) the scalar projection of $\underset{\sim}{p}$ onto $\underset{\sim}{q}$. [3]

$|p| = \sqrt{9+49}$ $\quad$ $|q| = \sqrt{4+36}$

$= \sqrt{58}$ $\quad$ $= \sqrt{40}$

$p \cdot q = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$= -6 + 42$

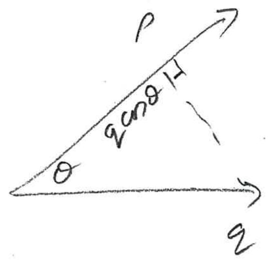
$= 36$

$p \cdot q = |p||q| \cos\theta$

$\frac{p \cdot q}{|q)} = |p| \cos\theta$

$\frac{36}{\sqrt{40}} = |p| \cos\theta$

(b) the vector projection of $\underset{\sim}{q}$ onto $\underset{\sim}{p}$. [3]



$p \cdot q = |p||q| \cos\theta$

$\frac{p \cdot q}{|p|} = |q| \cos\theta$

$\frac{p \cdot q}{|p|} \times \frac{p}{|p|} = |q| \cos\theta \, \hat{q}$

$\frac{36}{\sqrt{58}} \times \frac{3i + 7j}{\sqrt{58}} = |q| \cos\theta \, \hat{q}$

$\frac{36}{58}(3i + 7j) = |q| \cos\theta \, \hat{q}$

TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1,2
Test 2 2021

Section 2 Calculator Assumed
Vectors

STUDENT'S NAME ______________________________

DATE: Wednesday 31st March **TIME:** 28 minutes **MARKS:** 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (7 marks)

Given $\underset{\sim}{a} = \begin{pmatrix} -6 \\ 5 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} -2 \\ -8 \end{pmatrix}$

(a) calculate $|\underset{\sim}{a}|$ [1]

$\sqrt{61}$

(b) determine the unit vector parallel to $2\underset{\sim}{a} - \underset{\sim}{b}$ [3]

$2\begin{pmatrix} -6 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ -8 \end{pmatrix} = \begin{pmatrix} -10 \\ 18 \end{pmatrix}$

$\left|\begin{matrix} -10 \\ 18 \end{matrix}\right| = \sqrt{424}$

REQ'D VECTOR $= \dfrac{-10i + 18j}{\sqrt{424}}$

(c) determine the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ [3]

$a \cdot b = |a||b| \cos\theta$

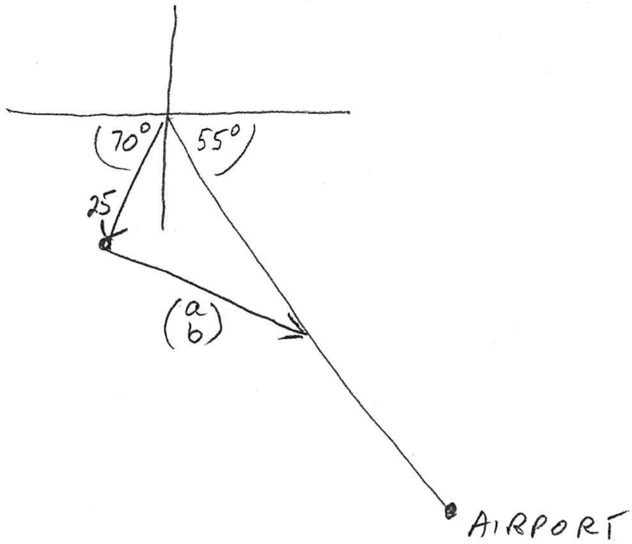
$-28 = \sqrt{61}\sqrt{68} \cos\theta$

$\theta = 115.8°$

6. (10 marks)

Johann is planning to fly his aircraft to an airport 370 km away on a bearing of 145°. His aircraft has a speed of 240 km/hr in still air. Throughout the flight there is an expected wind of speed 25 km/hr from a bearing of 020°.

(a) Show this information on a diagram. [2]



(b) Determine the setting on the aircraft in order to travel directly to the airport. [5]

$$\begin{pmatrix} 370 & \angle -55 \end{pmatrix} = \begin{pmatrix} 212.22 \\ -303.09 \end{pmatrix}$$

$$\begin{pmatrix} 25 & \angle -110 \end{pmatrix} = \begin{pmatrix} -8.55 \\ -23.49 \end{pmatrix}$$

$$\begin{pmatrix} -8.55 \\ -23.49 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} 212.22 \\ -303.09 \end{pmatrix}$$

$-8.55 + a = 212.22\lambda$ $\quad a = 212.22\lambda + 8.55$

$-23.49 + b = -303.09\lambda$ $\quad b = -303.09\lambda + 23.49$

$$a^2 + b^2 = 240^2$$

$$\lambda = 0.685$$

$$a = 153.92$$

$$b = -184.12$$

(c) Determine the actual ground speed of the aircraft. [2]

$\lambda \times 370$

$= 253.45$ km/hr

(d) Determine the time taken for the flight. [1]

$\frac{1}{\lambda} = 1.46$ hrs.

7. (5 marks)

Points O, P, Q and R have position vectors $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 15 \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

(a) Determine the value of y if $|OP| = 17$. [2]

$OP = \begin{pmatrix} 15 \\ y \end{pmatrix}$

$\sqrt{15^2 + y^2} = 17^2$

$y = \pm 8$

(b) Determine the value of x if $\overrightarrow{OP} \perp \overrightarrow{QR}$ using the value of y from (a) [3]

$\overrightarrow{QR} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} x \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 3-x \\ -4 \end{pmatrix}$

$\begin{pmatrix} 15 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 3-x \\ -4 \end{pmatrix} = 0$

$45 - 15x - 32 = 0$

$13 = 15x$

$\frac{13}{15} = x$

$\begin{pmatrix} 15 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 3-x \\ -4 \end{pmatrix} = 0$

$45 - 15x + 32 = 0$

$77 = 15x$

$\frac{77}{15} = x$

8. (6 marks)

Two forces act on a body. The first has magnitude 250 N and acts in the direction (-120°). The second force acts in the direction 170° and has a magnitude 410 N.

(a) By writing each force in component form, determine the resultant of the two forces. [3]

$F_1 = \begin{pmatrix} -125 \\ -216.51 \end{pmatrix}$ $F_2 = \begin{pmatrix} -403.77 \\ 71.2 \end{pmatrix}$

$F_1 + F_2 = \begin{pmatrix} -528.77 \\ -145.31 \end{pmatrix}$

(b) The work done, in Joules, by a force in moving a body is the scalar (dot) product of the resultant force from (a) in Newtons and the displacement in metres. Determine the work done by the two forces if the body moves 45 metres in the direction 215°.
Hint: write the displacement in component form. [3]

$(45 \angle 215°) = \begin{pmatrix} -36.86 \\ -25.81 \end{pmatrix}$

$\begin{pmatrix} -528.77 \\ -145.31 \end{pmatrix} \cdot \begin{pmatrix} -36.86 \\ -25.81 \end{pmatrix} = 23240.9 \text{ J}$